

NEUTRINO MASSES IN SUPERSYMMETRIC THEORIES**George K. Leontaris^a and Smaragda Lola^{a,b}**^a Department of Physics, Ioannina University, Ioannina, Greece^b Institut für Theoretische Physik, Universität Heidelberg, Germany

(Talk given by S. Lola at the 5th Hellenic School and Workshops on Elementary Particle Physics, Corfu, September 1995).

ABSTRACT

Extensions of the Standard Model with additional $U(1)$ symmetries can describe the hierarchy of fermion masses and mixing angles, including neutrinos. The neutrino masses and mixings are determined up to a discrete ambiguity corresponding to the representation content of the Higgs sector responsible for the Majorana mass matrix. The solar and the atmospheric neutrino deficits as well as the COBE data, may be explained simultaneously in specific schemes motivated by symmetries. Using simple, analytic expressions, it is possible to demonstrate the known effect that for small $\tan\beta$, phenomenologically interesting neutrino masses would disturb the bottom-tau unification. This however can be avoided in schemes with a large $\mu - \tau$ mixing in the charged leptonic sector. On the other hand, for the large $\tan\beta$ regime, due to the fixed point properties of the top as well as the bottom coupling (which are described by analytic expressions, for sufficiently large couplings) no modification to the bottom-tau unification would occur. Still, large mixing in the $\mu - \tau$ sector is desirable in this case as well, in order to have a solution to the atmospheric neutrino problem. In the same schemes, a relatively heavy strange quark ≈ 200 MeV is also predicted.

1 Introduction

Although the Standard Model describes successfully the strong and electroweak phenomena, there are still unanswered questions, related to the origin of fermion masses and mixing angles. An obvious possibility is that some symmetry, additional to that of the Standard Model is responsible for the pattern of masses and mixings that we see at low energies. And although unification on its own does not agree with experiment, when combined with supersymmetry it leads to very successful predictions for the gauge couplings, the pattern and magnitude of spontaneous symmetry breaking at the electroweak scale and the bottom - tau ($b - \tau$) unification. A further indication that additional symmetries beyond the Standard Model exist, has been the observation that the fermion mixing angles and masses have values consistent with the appearance of “texture” zeros in the mass matrices [1, 2].

On the other hand, neutrino data from various experiments seems to require certain mixings between various types of massive neutrinos. For these unknown neutrino masses and mixings, a similar hierarchy as the one for quarks and leptons may be expected to hold. The picture coming from the experiments is the following: The solar neutrino puzzle can be resolved through matter enhanced or vacuum oscillations with

$$\begin{aligned} \sin 2\theta_{ex} &= (0.39 - 1.6) \times 10^{-2}, & \Delta m^2 &= (0.6 - 1.2) \times 10^{-5} eV^2 \\ \sin 2\theta_{ex} &\geq 0.75, & \Delta m^2 &= (0.5 - 1.1) \times 10^{-5} eV^2 \end{aligned} \quad (1)$$

respectively [3]. On the other hand, the atmospheric neutrino problem may be explained in the case that large mixing and small mass splitting involving the muon neutrino exists [4]. Taking into account the bounds from accelerator and reactor disappearance experiments one finds that

$$\sin^2 2\theta_{\mu\alpha} \geq 0.51 - 0.6, \quad \delta m_{\nu_\alpha \nu_\mu}^2 \leq 10^{-2} \text{ eV}^2 \quad (2)$$

Finally, if neutrinos are to provide a hot dark matter component, as COBE requires [5], then the heavier neutrino(s) should have mass in the range $\sim (1 - 6) \text{ eV}$, the precise value depending on the number of neutrinos that have masses of this order of magnitude.

In what follows, we will discuss the expectations on these masses and mixings, from textures predicted by $U(1)$ symmetries.

2 Quark and Charged Lepton Masses

We start by reviewing the construction of the model of quark and charged lepton masses, that has been proposed by Ibanez and Ross [6]. The structure of the mass matrices is determined by a family symmetry, $U(1)_{FD}$, with the charge assignment of the Standard Model states given in Table 1. The need to preserve $SU(2)_L$ invariance requires left-handed up and down quarks (leptons) to have the same charge. This, plus the additional requirement of symmetric matrices, indicates that all quarks (leptons) of the same i -th generation transform with the same charge $\alpha_i(a_i)$. The full anomaly free Abelian group involves an additional family independent component, $U(1)_{FI}$, and with this freedom $U(1)_{FD}$ is made traceless without any loss of generality. Thus $\alpha_3 = -(\alpha_1 + \alpha_2)$ and $a_3 = -(a_1 + a_2)$.

If the light Higgs, H_2 , H_1 , responsible for the up and down quark masses respectively, have $U(1)$ charge so that only the (3,3) renormalisable Yukawa coupling to H_2 , H_1 is allowed, then only the (3,3) element of the associated mass matrix will be non-zero. The remaining entries are generated when the $U(1)$ symmetry is broken. This breaking is taken to be spontaneous via Standard Model singlet fields, θ , $\bar{\theta}$, with $U(1)_{FD}$ charge -1, +1 respectively, with equal vevs (vacuum expectation values). After this breaking the mass matrix acquires its structure. For example, the (3,2) entry in the up quark mass matrix appears at $O(\epsilon^{|\alpha_2 - \alpha_1|})$ because $U(1)$ charge conservation allows only a coupling $c^u t H_2 (\theta/M_2)^{\alpha_2 - \alpha_1}$, $\alpha_2 > \alpha_1$ or $c^d t H_2 (\bar{\theta}/M_2)^{\alpha_1 - \alpha_2}$, $\alpha_1 > \alpha_2$. Here $\epsilon = (\langle \theta \rangle / M_2)$ where M_2 is the unification mass scale which governs the higher dimension operators. A different scale, M_1 , is expected for the down quark mass matrices (we come back to this point below). Suppressing unknown Yukawa couplings and their phases (which are all expected to be of order unity), one arrives at mass matrices of the form

$$\frac{M_u}{m_t} \approx \begin{pmatrix} \epsilon^{|2+6a|} & \epsilon^{|3a|} & \epsilon^{|1+3a|} \\ \epsilon^{|3a|} & \epsilon^2 & \epsilon^1 \\ \epsilon^{|1+3a|} & \epsilon^1 & 1 \end{pmatrix}, \quad \frac{M_d}{m_b} \approx \begin{pmatrix} \bar{\epsilon}^{|2+6a|} & \bar{\epsilon}^{|3a|} & \bar{\epsilon}^{|1+3a|} \\ \bar{\epsilon}^{|3a|} & \bar{\epsilon}^2 & \bar{\epsilon}^1 \\ \bar{\epsilon}^{|1+3a|} & \bar{\epsilon}^1 & 1 \end{pmatrix} \quad (3)$$

where $\bar{\epsilon} = (\frac{\langle \bar{\theta} \rangle}{M_1})^{|\alpha_2 - \alpha_1|}$, $\epsilon = (\frac{\langle \theta \rangle}{M_2})^{|\alpha_2 - \alpha_1|}$ and $a = \alpha_1/(\alpha_2 - \alpha_1)$. In this simplest realisation, $h_b \approx h_t$ therefore we are in the large $\tan \beta$ regime of the parameter space of the MSSM. However, for a different H_1 and H_2 charge assignment, or in the presence of a mixing with additional Higgs fields, with the same $U(1)$ quantum numbers, it is possible to generate different h_b and h_t couplings, thus allowing for *any value of* $\tan \beta$. With $a = 1$, $\sqrt{\epsilon} = \bar{\epsilon} = 0.23$, (implying that $M_2 > M_1$), the matrices are in excellent agreement with the measured values of the quark masses. This relation for ϵ and $\bar{\epsilon}$ will also be very helpful below, in order to determining the neutrino mass spectrum.

The charged lepton mass matrix is determined in a similar way. Requiring $m_b = m_\tau$ at unification, sets $\alpha_1 = a_1$ and then the charged lepton mass matrix is

$$\frac{M_l}{m_\tau} \approx \begin{pmatrix} \bar{\epsilon}^{|2+6a-2b|} & \bar{\epsilon}^{|3a|} & \bar{\epsilon}^{|1+3a-b|} \\ \bar{\epsilon}^{|3a|} & \bar{\epsilon}^{|2(1-b)|} & \bar{\epsilon}^{|1-b|} \\ \bar{\epsilon}^{|1+3a-b|} & \bar{\epsilon}^{|1-b|} & 1 \end{pmatrix} \quad (4)$$

where $b = (\alpha_2 - a_2)/(\alpha_2 - \alpha_1)$. From the two choices found in [6] to lead to reasonable lepton masses,

the one which in principle leads to the maximum mixing in the $\mu - \tau$ is the choice $b = 1/2$ ¹. We will come back to this point at a latter stage.

3 First predictions for neutrino masses from symmetries

The first step in an effort to describe neutrino masses, is to determine the Dirac and heavy Majorana mass matrices. Here, we look at what happens in the case we add three generations of right-handed neutrinos, which will lead for predictions for light neutrino masses through the See-Saw mechanism. In such a scheme, the light Majorana neutrino masses are given by

$$M_{\nu}^{eff} = M_{\nu}^D \cdot (M_{\nu_R}^M)^{-1} \cdot M_{\nu}^D \quad (5)$$

$SU(2)_L$ fixes the $U(1)_{FD}$ charge of the left-handed neutrino states to be the same as the charged leptons. The left- right- symmetry then fixes the charges of the right-handed neutrinos as given in Table 1 and therefore the neutrino Dirac mass is

$$\frac{M_{\nu_R}^D}{m_{\nu\tau}} \approx \begin{pmatrix} \epsilon^{|2+6a-2b|} & \epsilon^{|3a|} & \epsilon^{|1+3a-b|} \\ \epsilon^{|3a|} & \epsilon^{|2(1-b)|} & \epsilon^{|1-b|} \\ \epsilon^{|1+3a-b|} & \epsilon^{|1-b|} & 1 \end{pmatrix} \quad (6)$$

The scale of this mass matrix is the same as for the up-quark mass matrices, similar to models based on Grand Unified Theories.

Of course the mass matrix structure of neutrinos is more complicated, due to the possibility of Majorana masses for the right-handed components. These arise from a term of the form $\nu_R \nu_R \Sigma$ where Σ is a $SU(3) \otimes SU(2) \otimes U(1)$ invariant Higgs scalar field with $I_W = 0$ and ν_R is a right-handed neutrino. The possible choices for the Σ $U(1)_{FD}$ charge will give a discrete spectrum of possible forms for the Majorana mass, M_{ν}^M [7, 8]. For example if, in the absence of $U(1)_{FD}$ symmetry breaking, the Σ charge is the same as the $H_{1,2}$ doublet Higgs charges, only the (3,3) element of M_{ν} will be non-zero. Allowing for $U(1)_{FD}$ breaking by $\langle \theta \rangle$ and $\langle \bar{\theta} \rangle$ the remaining elements in the Majorana mass matrix will be generated in an analogous way to the generation of the Dirac mass matrices².

An important point is to determine the appropriate expansion parameter and this brings us back to the generation of the scales M_1 and M_2 . Consider a string compactification, which, besides H_1 and H_2 , leaves additional Higgs multiplets $H_{1,2}^{a,b,\dots}, \bar{H}_{1,2}^{a,b,\dots}$ light. The pairs of Higgs fields in conjugate representations are expected to acquire gauge invariant masses, if there is any stage of spontaneous symmetry breaking at a scale M below the compactification scale, where $M = \langle \Phi \rangle$ and Φ is a gauge invariant combination of Higgs fields. However, there may be further sources of Higgs mass. The left-right symmetry, essentially requires an extension of the gauge symmetry to $SU(2)_L \otimes SU(2)_R$ at high scales. This will be broken by a right-handed sneutrino vev in which case the mass degeneracy of the H_1 and H_2 pair which transform as a $(1/2, 1/2)$ representation under $SU(2)_L \otimes SU(2)_R$ can be split via the coupling $\langle \bar{\nu}_R \rangle H_2 H_x$ where H_x transforms as $(1/2, 0)$. Such a contribution will generate $M_2 \approx \langle \bar{\nu}_R \rangle$, $M_1 \approx M$. Then, one expects that the $\bar{\nu}_R$ fields acquire a mass of $O(M_1)$ via a $\Phi \nu \bar{\nu}$ coupling, implying that the appropriate expansion parameter for the Majorana mass matrix is the same as that for the *down* quarks and charged leptons [7].

We may now compute the patterns of Majorana mass for the different possible choices of Σ charge. These are given in Table 2 [8]. For $\alpha = 1$, $\beta \equiv 1 - b = 1/2$, we can obtain the specific forms for Dirac and Majorana textures compatible with the correct fermion mass predictions in the presence of the intermediate neutrino scale. In Table 3 we present the eigenvalues of the heavy Majorana mass matrix for this choice of α and β . The eigenvalues of m_{eff} are given in Table 4. The order of the matrices in Tables 2 and 3 corresponds to the one of Table 2.

¹ In [6] a residual Z_2 discrete gauge symmetry after $U(1)$ breaking by which the electron and muon fields get transformed by a factor (-1) , was imposed. This resulted in entries raised in a half-integer power being set to zero, eliminating the (2,3) entries in the mass matrix at the GUT scale. However this is not a necessary condition and once the Z_2 symmetry is dropped, the relevant (2,3) entries may be quite large.

² $\langle \Sigma \rangle$ is significantly below the Planck scale and thus $\langle \theta \rangle$ dominates the $U(1)_{FD}$ breaking.

The first point to note from these structures, is that in none of the cases does the light Majorana mass matrix have degenerate eigenvalues, which in the past had been the most common assumption. This occurs because the charges of the right-handed neutrinos force the mass matrix entries to be of different orders in powers of the expansion parameter ϵ . In the case where two components are coupled through an off diagonal mass term as in cases 2, 4, 5 and 9, two out of the three eigenvalues may be approximately equal. For the light Majorana neutrino masses, the structure of the Dirac mass matrices results in an even larger spread. From the values quoted in the introduction, we see that in order to solve the solar, atmospheric and dark matter problems simultaneously, three nearly degenerate neutrinos of approximate mass $1 - 2$ eV are required [9]. This is not the case in the simplest scheme that we have been discussing, without fine tuning. We will see below a more complicated scheme, with more than one Σ fields, where this becomes possible. Before doing so, however, let us consider the rest of the implications that this simplest scheme has.

Besides the relative strength of the neutrino masses, we would also like to know what are the expectations for their absolute magnitudes. This depends on the origin of the field Σ . If $\Sigma = \tilde{\nu}_R \tilde{\nu}_R$ then the Majorana masses are expressed in units $\langle \tilde{\nu}_R \rangle \langle \tilde{\nu}_R \rangle / M_c$ where M_c is the mass scale governing the appearance of higher dimension operators, typically the string scale or M_{Planck} . For a unification scale $O(10^{16} GeV)$ it is reasonable to choose $\langle \tilde{\nu}_R \rangle = O(10^{16} GeV)$ leading to a scale $10^{13} - 10^{14} GeV$ for the Majorana mass scale. Then the mass unit for the light neutrinos is roughly $O(4 - 0.4) eV$ for a top quark of $O(200) GeV$ [7], which is an interesting feature. An additional interesting point is that the mixing in the (2,3) entries is of the correct order of magnitude for a possible solution to the atmospheric neutrino problem³. Indeed, the diagonalising matrix is given by

$$V \approx \begin{pmatrix} \sqrt{1 - \bar{\epsilon}^2} & \bar{\epsilon} \\ -\bar{\epsilon} & \sqrt{1 - \bar{\epsilon}^2} \end{pmatrix} \quad (7)$$

Then, the textures of Table 4 indicate towards two possible solutions: In solution (A), one may fit the COBE results and solve the solar neutrino problem, while in solution (B) [8], it is possible to obtain a simultaneous solution to the solar and the atmospheric neutrino problems. Whether we obtain the solution (A) or (B) depends on the predicted mass splitting between the two heavy neutrinos in each of the six choices for the heavy Majorana mass matrix. For a $\nu_\tau \approx 5$ eV, we obtain a muon neutrino mass $m_{\nu_\mu} = m_{\nu_\tau} x_i = 0.06, 0.014$ and 0.003 eV respectively, where $x_i \equiv e^6, e^8$ and e^{10} , is the splitting between the two larger eigenvalues of m_{eff} . This indicates that, the matrices with a total splitting e^{10} naturally lead to a solution of the COBE measurements and the solar neutrino problem. On the other hand, for $m_{\nu_\tau} \approx 0.1$ eV and $x_1 = e^6$, $m_{\nu_\mu} = m_{\nu_\tau} x_1 = 0.0012$ eV, which may be marginally consistent with a solution to the atmospheric and solar neutrino problems (remember that coefficients of order unity have not yet been defined in the solutions. This was recently done in [10], using infrared-fixed point arguments). Since there are alternative schemes which lead to an explanation of the COBE measurements, other than hot and cold dark matter⁴, we believe that the scheme (B) should be considered on equivalent grounds with the scheme (A), which has been discussed extensively in [7].

4 Solutions with three degenerate neutrinos

In the previous section, the simplest scheme with a $U(1)$ symmetry has been considered and while two classes of solutions were found, it has not been possible to solve all three neutrino problems at the same time. This problem is expected to disappear, once we go to schemes with more than one Σ fields. However in this case the possible choices one can make increase a lot. For this reason, instead of searching a priori for a more sophisticated model that may accommodate the experimental data, we will follow the opposite procedure [15]: We first consider models that potentially allow the consistent

³ The mixing in the (1,2) sector is negligible.

⁴ For example, we have found that domain walls may give structure at medium and large scales if, either they are unstable, or the minima of the potentials of the relevant scalar field appear with different probabilities [11].

incorporation of all experimental data and look at the form that the heavy Majorana mass matrix should have, and then see how this mass matrix arises from symmetries. To do so, we initially assume a strong mixing in the 2-3 entries of the effective mass matrix. This will then enable a solution of the atmospheric neutrino problem. To simplify the analysis, we take the 1-2 and 1-3 mixing angles to be zero in this simple example, assuming that the MSW oscillations are generated by the charged current interactions, as in [12]. Furthermore we take three nearly degenerate masses. From $M_{\nu_R} = m_D^\dagger \cdot m_{eff}^{-1} \cdot m_D$ and with the mixing matrix

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_1 & -s_1 \\ 0 & s_1 & c_1 \end{pmatrix} \quad (8)$$

$m_{eff}^{-1} = V_\nu m_{eff}^{-1diag} V_\nu^T$ will have the form

$$m_{eff}^{-1} = \begin{pmatrix} \frac{1}{m_1} & 0 & 0 \\ 0 & \frac{c_1^2}{m_2} + \frac{s_1^2}{m_3} & c_1 s_1 \left(\frac{1}{m_2} - \frac{1}{m_3} \right) \\ 0 & c_1 s_1 \left(\frac{1}{m_2} - \frac{1}{m_3} \right) & \frac{c_1^2}{m_3} + \frac{s_1^2}{m_2} \end{pmatrix} \equiv \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & d & c \end{pmatrix}. \quad (9)$$

where m_i are the eigenvalues of m_{eff} . Identifying the entries gives

$$\begin{aligned} \sin^2 2\theta_1 &= \frac{4d^2}{(m_2^{-1} - m_3^{-1})^2} \\ m_1^{-1} &= a \\ m_2^{-1} &= \frac{b}{2} + \frac{c}{2} + \frac{1}{2} \sqrt{b^2 - 2bc + c^2 + 4d^2} \\ m_3^{-1} &= \frac{b}{2} + \frac{c}{2} - \frac{1}{2} \sqrt{b^2 - 2bc + c^2 + 4d^2}, \end{aligned} \quad (10)$$

where θ_1 is the $\mu - \tau$ mixing angle. The case of the absolute value of the three masses equal (i.e. $m_1 = m_2, m_3 = -m_2$ is equivalent to $b = c = 0, a = d$, therefore $\sin^2 2\theta_1 = 1, \theta = 45^\circ$.

Subsequently, we assume the very large class of models from underlying unified models (such as strings and grand unified theories, or partially unified models) which fix the neutrino Dirac mass matrix to be proportional to the u-quark mass matrix. For example, the form of the heavy Majorana mass matrix corresponding to an up-quark mass matrix of the form [13, 14]

$$m_\nu^D = \begin{pmatrix} 0 & 0 & x \\ 0 & x & 0 \\ x & 0 & 1 \end{pmatrix}, \quad (11)$$

is given by

$$\begin{pmatrix} x^2 \left(\frac{c_1^2}{m_3} + \frac{s_1^2}{m_2} \right) & x^2 \frac{\sin 2\theta_1}{2} \left(\frac{1}{m_2} - \frac{1}{m_3} \right) & x \left(\frac{c_1^2}{m_3} + \frac{s_1^2}{m_2} \right) \\ x^2 \frac{\sin 2\theta_1}{2} \left(\frac{1}{m_2} - \frac{1}{m_3} \right) & x^2 \left(\frac{c_1^2}{m_2} + \frac{s_1^2}{m_3} \right) & x \frac{\sin 2\theta_1}{2} \left(\frac{1}{m_2} - \frac{1}{m_3} \right) \\ x^2 \left(\frac{c_1^2}{m_3} + \frac{s_1^2}{m_2} \right) & x \frac{\sin 2\theta_1}{2} \left(\frac{1}{m_2} - \frac{1}{m_3} \right) & \frac{x^2}{m_1} + \frac{c_1^2}{m_3} + \frac{s_1^2}{m_2} \end{pmatrix}. \quad (12)$$

For the above values of the three masses this becomes [15]

$$M_{\nu_R} = \begin{pmatrix} 0 & M_N x & 0 \\ M_N x & 0 & M_N \\ 0 & M_N & M_N x \end{pmatrix}, \quad (13)$$

where $M_N = xd \approx 10^{11} - 10^{13}$ GeV. So we see that in this example the degeneracy of all three masses and one large mixing angle is consistent and may be understood in terms of texture zeroes of the heavy Majorana neutrino mass matrix M_{ν_R} .

A systematic study of such solutions has been carried in [15], where all possible cases with at least one large mixing angle are given. The quoted mass matrices may arise due to symmetries, when more than one Σ and θ fields are present. To see how this occurs, let us note the following: Assume the existence of a Σ field with a charge -1 , which makes the (2,3) entry unity. This leads to the relevant heavy Majorana mass matrices that we have already derived. Suppose now that a second Σ exists, with quantum number $+2$. This means that from the original matrix, the dominant element will be the one with the biggest absolute power in $\bar{\epsilon}$ ie, the elements (2,2), (2,3) and (3,3) still would couple to Σ_1 with charge -1 , while the (1,2) and (1,3) will couple to Σ_2 . Then the matrix will be

$$M_u \approx \begin{pmatrix} 0 & \epsilon^{-3+2} & \epsilon^{-4+2} \\ \epsilon^{-3+2} & \epsilon & 1 \\ \epsilon^{-4+2} & 1 & \epsilon^{-1} \end{pmatrix} \quad (14)$$

This structure is similar to that of the example we just gave, with the difference that the (2,2) element is of order ϵ . However this does not affect the predictions [15], since it results to a small deviation from the picture that we have discussed.

5 RGE with RH-neutrinos: an analytic approach

From the above it is clear that the interpretation of many important experimental facts is based on the existence of the right – handed partners ν_{R_i} of the three left – handed neutrinos, where the scale of mass of these particles is at least three orders of magnitude smaller than the gauge unification scale, M_U . Thus the running from the unification scale, $M_U \sim 10^{16}$ GeV, down to the scale of M_{ν_R} , must include radiative corrections from ν_R neutrinos. After that scale, ν_R 's decouple from the spectrum, and an effective see – saw mechanism is operative, c.f. eq(5). In the presence of the right handed neutrino, the renormalization group equations for the Yukawa couplings at the one-loop level, for the small $\tan \beta$ regime, where only the top and Dirac – type neutrino Yukawa couplings are large at the GUT scale, can be written in a diagonal basis as follows [16]

$$16\pi^2 \frac{d}{dt} h_t = (6h_t^2 + h_N^2 - G_U) h_t, \quad (15)$$

$$16\pi^2 \frac{d}{dt} h_N = (4h_N^2 + 3h_t^2 - G_N) h_N, \quad (16)$$

$$16\pi^2 \frac{d}{dt} h_b = (h_t^2 - G_D) h_b, \quad (17)$$

$$16\pi^2 \frac{d}{dt} h_\tau = (h_N^2 - G_E) h_\tau, \quad (18)$$

Here, h_α , $\alpha = U, D, E, N$, represent the $3 \otimes 3$ Yukawa matrices for the up and down quarks, charged lepton and Dirac neutrinos, while I is the $3 \otimes 3$ identity matrix. Finally, $G_\alpha = \sum_{i=1}^3 c_\alpha^i g_i(t)^2$ are functions which depend on the gauge couplings with the coefficients c_α^i 's given by [12, 17].

$$\{c_U^i\}_{i=1,2,3} = \left\{ \frac{13}{15}, 3, \frac{16}{3} \right\}, \quad \{c_D^i\}_{i=1,2,3} = \left\{ \frac{7}{15}, 3, \frac{16}{3} \right\}, \quad (19)$$

$$\{c_E^i\}_{i=1,2,3} = \left\{ \frac{9}{5}, 3, 0 \right\}, \quad \{c_N^i\}_{i=1,2,3} = \left\{ \frac{3}{5}, 3, 0 \right\}. \quad (20)$$

Below M_N , the right handed neutrino decouples from the massless spectrum and we are left with the standard spectrum of the MSSM. For scales $Q \leq M_N$ the gauge and Yukawa couplings evolve according to the standard renormalisation group equations. To gain an insight into the effects of new couplings associated with the ν_R in the renormalisation group running we integrate the above equations in the region $M_N \leq Q \leq M_U$. We denote the top and ν_R Yukawas at the unification scale by h_G , while the bottom and τ couplings are denoted with h_{b_0}, h_{τ_0} respectively. The top and neutrino Yukawa couplings

at the unification scale are equal, a relation which arises naturally not only in our case but in most of the Grand Unified Models which predict the existence of right handed neutrinos. Then [8]

$$h_t(t) = \gamma_U(t) h_G \xi_t^6 \xi_N \quad (21)$$

$$h_N(t) = \gamma_N(t) h_G \xi_t^3 \xi_N^4 \quad (22)$$

$$h_b(t) = \gamma_D(t) h_{b_0} \xi_t \quad (23)$$

$$h_\tau(t) = \gamma_E(t) h_{\tau_0} \xi_N \quad (24)$$

where the functions $\gamma_\alpha(t)$ and ξ_i depend purely on gauge coupling constants and Yukawa couplings respectively, and are given by

$$\begin{aligned} \gamma_\alpha(t) &= \exp\left(\frac{1}{16\pi^2} \int_{t_0}^t G_\alpha(t) dt\right) = \prod_{j=1}^3 \left(\frac{\alpha_{j,0}}{\alpha_j}\right)^{c_\alpha^j/2b_j} \\ \xi_i &= \exp\left(\frac{1}{16\pi^2} \int_{t_0}^t \lambda_i^2 dt\right) \end{aligned} \quad (25)$$

One then finds that

$$h_b(t_N) = \rho \xi_t \frac{\gamma_D}{\gamma_E} h_\tau(t_N) \quad (26)$$

with $\rho = \frac{h_{b_0}}{h_{\tau_0} \xi_N}$. In the case of $b - \tau$ unification at M_U , we have $h_{\tau_0} = h_{b_0}$, while in the absence of the right - handed neutrino $\xi_N \equiv 1$, thus $\rho = 1$ and the m_b mass has the phenomenologically reasonable prediction at low energies. In the presence of ν_R however, if $h_{\tau_0} = h_{b_0}$ at the GUT scale, the parameter ρ is no longer equal to unity since $\xi_N < 1$. In fact the parameter ξ_N becomes smaller for lower M_N scales. Therefore, in order to restore the correct m_b/m_τ prediction at low energies we need $\rho = 1$ corresponding to $h_{b_0} = h_{\tau_0} \xi_N$. For $M_N \approx 10^{13} \text{ GeV}$ for example and $h_G \geq 1$, we can estimate that $\xi(t_N) \approx 0.89$ thus, there is a corresponding $\sim 10\%$ deviation of the $\tau - b$ equality at the GUT scale [8], in agreement with the numerical results of [17].

In the case of a large $\tan\beta$, a first thing to note is that there are important corrections to the bottom mass from one-loop graphs involving supersymmetric scalar masses and the μ parameter, which can be of the order of $(30 - 50)\%$ [18]. Moreover, even if one ignores these corrections, the effect of the heavy neutrino scale is much smaller, since now the bottom Yukawa coupling also runs to a fixed point, therefore its initial value does not play an important role. For example, for large $\tan\beta$, and $h_b \approx h_t$, the product and ratio of the top and bottom couplings, has been found in [19] to be

$$h_t h_b \approx \frac{8\pi^2 \gamma_Q \gamma_D}{7 \int \gamma_Q^2 dt}, \quad \frac{h_t^2}{h_b^2} \approx \frac{\gamma_Q^2}{\gamma_D^2} \quad (27)$$

indicating that one gets an approximate, model independent prediction for both couplings at the low energy scale. To see the effect of the neutrino scale to the $b - \tau$ unification in this case, we solved numerically the renormalisation group equations. In the small $\tan\beta$ regime, there exists a parameter space where the initial condition $h_t = 2.0$ and $h_b = 0.0125$ lead to a factor $\xi_N = 0.86$, for $M_N = 10^{12} \text{ GeV}$ and an upper limit for the running bottom mass $m_b = 4.35$. For the same parameter space, when we set $h_b = 2.0$, ξ becomes $\xi_N = 0.96$. Moreover, again for the same example, if we allow for a running bottom mass $m_b = 4.4$, $\xi_N = 0.99$. For higher heavy neutrino scales, the relevant effect is even smaller.

6 Restoration of bottom – tau unification

Given the results of the previous section, it is natural to ask if Grand Unified models which predict the $b - \tau$ equality at the Unification scale, exclude the experimentally required and cosmologically interesting region for the neutrino masses in the small $\tan\beta$ regime. To answer this question, we should first recall that the $b - \tau$ equality at the GUT scale refers to the $(3, 3)$ entries of the corresponding charged lepton and down quark mass matrices. The detailed structure of the mass matrices is not predicted, at least

by the Grand Unified Group itself, unless additional structure is imposed. It is possible then to assume $(m_E^0)_{33} = (m_D^0)_{33}$ and a specific structure of the corresponding mass matrices such that after the diagonalisation at the GUT scale, the $(m_E^\delta)_{33}$ and $(m_D^\delta)_{33}$ entries are no-longer equal [8]⁵.

To illustrate this point, let us present here a simple 2×2 example [8]. Assume a diagonal form of m_D^0 at the GUT scale, $m_D^0 = \text{diagonal}(cm_0, m_0)$, while the corresponding entries of charged lepton mass matrix have the form

$$m_E^0 = \begin{pmatrix} d & \tilde{\epsilon} \\ \tilde{\epsilon} & 1 \end{pmatrix} m_0 \quad (28)$$

These forms of m_D^0, m_E^0 ensure that at the GUT scale $(m_D^0)_{33} = (m_E^0)_{33}$. However, at low energies one should diagonalize the renormalised Yukawa matrices to obtain the correct eigenmasses. Equivalently, one can diagonalise the quark and charged lepton Yukawa matrices at the GUT scale and evolve separately the eigenstates and the mixing angles. Since m_D^0 has been chosen diagonal, the mass eigenstates which are to be identified with the s, b - quark masses at low energies are given by $m_s = c\gamma_D m_0$ and $m_b = \gamma_D m_0 \xi_t$, with $m_0 = h_{b_0} \frac{v}{\sqrt{2}} \cos\beta$. To find the charged lepton mass eigenstates we need first to diagonalise m_E^0 at M_{GUT} . We can obtain the following relations between the entries $\tilde{\epsilon}, d$ of m_E^0 and the mass eigenstates m_μ^0, m_τ^0 at the GUT scale

$$d = \left(\frac{m_\tau^0 - m_\mu^0}{m_0} - 1 \right), \quad \tilde{\epsilon}^2 = \left(\frac{m_\mu^0}{m_0} + 1 \right) \left(\frac{m_\tau^0}{m_0} - 1 \right) \quad (29)$$

In the presence of right handed neutrinos, the evolution of the above τ - eigenstate down to low energies is described by (18) with $m_{\tau_0} = h_{\tau_0} \frac{v}{\sqrt{2}} \cos\beta$. By simple comparison of the obtained formulae, we conclude that, to obtain the correct m_τ/m_b ratio at m_W while preserving the $b - \tau$ unification at m_{GUT} , the m_E^0 entries should satisfy the following relations

$$\tilde{\epsilon} = \sqrt{\frac{1}{\xi_N} - 1}, \quad d \approx \left(\frac{1}{\xi_N} - 1 \right) = \tilde{\epsilon}^2 \quad (30)$$

The above result deserves some discussion. Firstly we see that it is possible to preserve $b - \tau$ unification by assuming 2 - 3 generation mixing in the lepton sector, even if the effects of the ν_R states are included. Secondly, this mixing is related to a very simple parameter which depends only on the scale M_N and the initial h_N condition. The range of the coefficient c in the diagonal form of the m_D^0 - matrix, can also be estimated using the experimental values of the quark masses m_s, m_b . An interesting observation is that the usual GUT - relation for the $(2, 2)$ - matrix elements of the charged lepton and down quark mass matrices, i.e., $(m_E)_{22} = -3(m_D)_{22}$, which in our case is satisfied for $c = -d/3$, implies here a relatively heavy strange quark mass $m_s \sim 200$ MeV. Smaller m_s values are obtained if $-3c/d < 1$ [8].

7 Conclusions

We have looked at the implications for neutrino masses and mixings, coming from $U(1)$ symmetries, in addition to the Standard Model gauge group. We find that it is possible to explain the solar, the atmospheric and the dark matter problems at the same time, in schemes which can be derived from such symmetries. Moreover, we have derived analytic expressions to describe the fact that in the small $\tan\beta$ regime, an intermediate neutrino scale would result to deviations from the bottom-tau unification (in the large $\tan\beta$ regime, one notices that due to the top and bottom coupling fixed point properties, no modification to the bottom-tau unification would occur). We proposed schemes where this deviation is avoided, by considering a large $\mu - \tau$ mixing in the charged leptonic sector. A relatively heavy strange quark ≈ 200 MeV is also predicted in the framework of these models.

⁵ An alternative solution occurs in a class of models where the symmetries lead to a neutrino Yukawa coupling much smaller than the top one [20].

References

- [1] The literature on the subject is vast. Some of the many references are: H. Fritzsch, Phys. Lett. **70B** (1977) 436; **B73** (1978) 317; Nucl. Phys. **B155** (1979) 189; C. D. Froggatt and H. B. Nilsen, Nucl. Phys. **B147** (1979) 277; J. Harvey, P. Ramond and D. Reiss, Phys. Lett. **B92** (1980) 309; C. Wetterich, Nucl. Phys. **B261** (1985) 461; P. Kaus and S. Meshkov, Mod. Phys. Lett. **A3** (1988) 1251; F.J. Gilman and Y. Nir, Ann. Rev. Nucl. Part. Sci. **40** (1990) 213; S. Dimopoulos, L. J. Hall and S. Raby, Phys. Rev. Lett. **68** (1992) 1984; Phys. Rev. **D45** (1992) 4195; H. Arason, D. J. Castaño, P. Ramond and E. J. Piard, Phys. Rev. **D47** (1993) 232; G. K. Leontaris and N. D. Tracas, Phys. Lett. **B303** (1993) 50; K. Babu and Q. Shafi, hep-ph/9503313; C. D. Froggatt, hep-ph/9504323; R. D. Peccei and K. Wang, hep-ph/9509242; P. H. Frampton, IFP-718-UNC, hep-ph/9510260.
- [2] Y. Achiman and T. Greiner, Nucl. Phys. **B443** (1995) 3; P. Binetruy and P. Ramond, Phys. Lett. **B350** (1995) 49; P. Ramond, hep-ph/9506319; E. Papageorgiu, Z. Phys. **C64** (1994) 509; Z. Phys. **C65** (1995) 135; hep-ph/9504208; Y. Grossman and Y. Nir, Nucl. Phys. **B448** (1995) 30; C. H. Albright and S. Nandi, hep-ph/9505383; B.C. Allanach and S.F. King, hep-ph/9509205, SHEP-95-28 (1995).
- [3] See for example, L. Wolfenstein, Phys.Rev. **D17** (1978) 20; S. P. Mikheyev and A. Yu Smirnov, Yad. Fiz. **42** (1985) 1441; J. N. Bahcall and W.C. Haxton, Phys.Rev. **D40** (1989) 931; X. Shi, D. N. Schramm and J. N. Bahcall, Phys.Rev.Lett. **69** (1992) 717; P. I. Krastev and S. Petcov, Phys.Lett. **B299** (1993) 94; N. Hata and P. Langacker, Phys.Rev. **D50** (1994) 632 and references therein.
- [4] K. S. Hirata et al., Phys. Lett. **B280** (1992) 164; R. Becker-Szendy et al., Phys. Rev. **D46** (1992) 3720; Y. Fukuda et al., Phys. Lett. **B335** (1994) 237.
- [5] G. F. Smoot et al., Astrophys. J. Lett. **396** (1992), L1.
- [6] L.E. Ibáñez and G.G. Ross, Phys. Lett. **B332** (1994) 100.
- [7] H. Dreiner, G. K. Leontaris, S. Lola, G. G. Ross and C. Scheich, Nucl. Phys. **B346** (1995) 461.
- [8] “Heavy neutrino threshold effects in low energy phenomenology”, G. Leontaris, S. Lola and G. G. Ross, hep-ph/9505402, Nucl. Phys. **B454** (1995) 25.
- [9] S. Petcov and A. Smirnov, Phys.Lett. **B322** (1994) 109; D. O. Caldwell, R. N. Mohapatra, UCSB-HEP-94-03; hep-ph-9402231.
- [10] G. G. Ross, “Fermion mass predictions from infra-red fixed points”, CERN-TH-95-162, hep-ph/9507368.
- [11] S. Lola and G. G. Ross, Nucl. Phys. **B406** (1993) 452; Z. Lalak, S. Lola, B. Ovrut and G. G. Ross, Nucl. Phys. **B434** (1995) 675.
- [12] H. Dreiner, G. K. Leontaris and N. D. Tracas, Mod. Phys. Lett. **A9** (1993) 2099.
- [13] G. F. Giudice, Mod. Phys. Lett. **A7** (1992) 2429.
- [14] P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. **B406** (1993) 19.
- [15] G. Leontaris, S. Lola, C. Scheich and J. Vergados, “Phenomenological textures for neutrino mass matrices”, HD-THEP-95-29, hep-ph/9509351.
- [16] P. H. Chankowski and Z. Pluciennik, Phys. Lett. **B316** (1993) 312; K. Babu, C. N. Leung and J. Pantaleone, Phys. Lett. **B319** (1993) 191;
- [17] F. Vissani and A. Yu. Smirnov, Phys. Lett. **B341** (1994) 173; A. Brignole, H. Murayama and R. Rattazzi, Phys. Lett. **B335** (1994) 345.
- [18] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. **D 50**(1994)7048; M. Carena, M. Olechowski, S. Pokorski, and C. E. M. Wagner, Nucl.Phys. **B426** (1994) 269.

- [19] E. Floratos, G. Leontaris and S. Lola, “MSSM phenomenology in the large $\tan\beta$ regime”, hep-ph/9507402, to be published in Phys. Lett. B.
- [20] S. Dimopoulos and A. Pomarol, “Non-unified sparticle and particle masses in unified theories”, CERN-TH/95-44, hep-ph/9502397.

| | Q_i | u_i^c | d_i^c | L_i | e_i^c | ν_i^c | H_2 | H_1 |
|-------------|------------|------------|------------|-------|---------|-----------|--------------|--------------|
| $U(1)_{FD}$ | α_i | α_i | α_i | a_i | a_i | a_i | $-2\alpha_1$ | $-2\alpha_1$ |

Table 1: $U(1)_{FD}$ charges.

| | |
|--|--|
| $\begin{pmatrix} \bar{\epsilon}^{2 3\alpha+\beta } & \bar{\epsilon}^{3 \alpha } & \bar{\epsilon}^{3\alpha+\beta } \\ \bar{\epsilon}^{3\alpha } & \bar{\epsilon}^{2 \beta } & \bar{\epsilon}^{ \beta } \\ \bar{\epsilon}^{3\alpha+\beta } & \bar{\epsilon}^{ \beta } & 1 \end{pmatrix}$ | $\begin{pmatrix} \bar{\epsilon}^{3 2\alpha+\beta } & \bar{\epsilon}^{3\alpha+\beta } & \bar{\epsilon}^{3\alpha+2\beta } \\ \bar{\epsilon}^{3\alpha+\beta } & \bar{\epsilon}^{ \beta } & 1 \\ \bar{\epsilon}^{3\alpha+2\beta } & 1 & \bar{\epsilon}^{ \beta } \end{pmatrix}$ |
| $\begin{pmatrix} \bar{\epsilon}^{2 3\alpha+2\beta } & \bar{\epsilon}^{3\alpha+2\beta } & \bar{\epsilon}^{3 \alpha+\beta } \\ \bar{\epsilon}^{3\alpha+2\beta } & 1 & \bar{\epsilon}^{ \beta } \\ \bar{\epsilon}^{3 \alpha+\beta } & \bar{\epsilon}^{ \beta } & \bar{\epsilon}^{2 \beta } \end{pmatrix}$ | $\begin{pmatrix} \bar{\epsilon}^{3\alpha+\beta } & \bar{\epsilon}^{ \beta } & 1 \\ \bar{\epsilon}^{ \beta } & \bar{\epsilon}^{3 \alpha+\beta } & \bar{\epsilon}^{3 \alpha+2\beta } \\ 1 & \bar{\epsilon}^{3 \alpha+2\beta } & \bar{\epsilon}^{3\alpha+\beta } \end{pmatrix}$ |
| $\begin{pmatrix} 1 & \bar{\epsilon}^{3\alpha+2\beta } & \bar{\epsilon}^{3\alpha+\beta } \\ \bar{\epsilon}^{3\alpha+2\beta } & \bar{\epsilon}^{2 3\alpha+2\beta } & \bar{\epsilon}^{3 2\alpha+\beta } \\ \bar{\epsilon}^{3\alpha+\beta } & \bar{\epsilon}^{3 2\alpha+\beta } & \bar{\epsilon}^{2 3\alpha+\beta } \end{pmatrix}$ | $\begin{pmatrix} \bar{\epsilon}^{3\alpha+2\beta } & 1 & \bar{\epsilon}^{ \beta } \\ 1 & \bar{\epsilon}^{3\alpha+2\beta } & \bar{\epsilon}^{3\alpha+\beta } \\ \bar{\epsilon}^{ \beta } & \bar{\epsilon}^{3\alpha+\beta } & \bar{\epsilon}^{3\alpha } \end{pmatrix}$ |

Table 2: General forms of heavy Majorana mass matrix textures. The specific textures of the text arise for $\alpha = 1, \beta = 1/2$.

| | |
|---|--|
| $\begin{pmatrix} e^{10} & & \\ & e^2 & \\ & & 1 \end{pmatrix}$ | $\begin{pmatrix} e^{15} & & \\ & -1+e & \\ & & 1+e \end{pmatrix}$ |
| $\begin{pmatrix} e^{16} & & \\ & e^2 & \\ & & 1 \end{pmatrix}$ | $\begin{pmatrix} e^9 & & \\ & -1-e^2 & \\ & & 1+e^2 \end{pmatrix}$ |
| $\begin{pmatrix} e^{16} & & \\ & e^{14} & \\ & & 1 \end{pmatrix}$ | $\begin{pmatrix} e^6 & & \\ & -1-e^2 & \\ & & 1+e^2 \end{pmatrix}$ |

Table 3: Eigenvalues of Heavy Majorana mass matrix textures, for $\alpha = 1$ and $\beta = 1/2$

| | |
|---|---|
| $\begin{pmatrix} e^{26} & & \\ & e^{10} & \\ & & 1 \end{pmatrix}$ | $\begin{pmatrix} e^{25} & & \\ & e^9 & \\ & & 1/e \end{pmatrix}$ |
| $\begin{pmatrix} e^{24} & & \\ & e^8 & \\ & & 1/e^2 \end{pmatrix}$ | $\begin{pmatrix} e^{33} & & \\ & e^{13} & \\ & & 1/e^7 \end{pmatrix}$ |
| $\begin{pmatrix} e^{40} & & \\ & 1/e^8 & \\ & & 1/e^{14} \end{pmatrix}$ | $\begin{pmatrix} e^{32} & & \\ & e^6 & \\ & & 1/e^6 \end{pmatrix}$ |

Table 4: Eigenvalues of light Majorana mass matrix textures, for $\alpha = 1$ and $\beta = 1/2$